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REVIEW ARTICLE

Mathematical Models studying Crime Dynamics: A Review on Adoptd Approaches

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ABSTRACT

As crime is becoming a major issue over the last few decades in India, mathematical models came as a rescue to study and understand the dynamics of crime and to analyze the effect of control and preventive measures. This review aims at discussing several approaches to formulate such mathematical models, namely, economical approach, epidemic approach, predator-prey modeling approach and spatio-temporal approach. Strategies for combating financial crime are also examined. Through this review, it has been observed that study on the dynamics of crime against women is less considered. Such gap could be analyzed with the help of information obtained through the approaches discussed. The work of this paper could help crime analysts to predict the trends in the crime against women and prepare the optimal control and prevention policies.

KEYWORDS | mathematical modelling, crime against women, spatio-temporal

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INTRODUCTION

Crime is considered to be a wrong committed against the society as it shakes the social conscience and sense of societal stability and fearlessness. It is a law prohibited event, followed by prosecution of accused and later by punishment on conviction. Out of several criminal activities, some are listed as burglary, fraud, cybercrime, financial crime, domestic abuse, child abuse, robbery, rape and murder.

There is a social transmission view for the spread of crime. Crime can be considered a social epidemic process, which may infect people by means of behavior, messages, social structure and traditions or ideas. It is assumed to be spread by social contact which in turn depends on social environment including some factors like poverty, education level, religion, socio-cultural practices and inequality. Certain type of people can effectively broadcast their ideas and convince people to listen and follow suit, leading to the transmission of criminal activities.

The level and intensity of crime can vary from nation to nation. As per the statistics released by NCRB (National Crime Records Bureau), it was observed

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that crime in India is increasing at an alarming rate over recent years. In 2016, 48.3 lakh crime cases were recorded, which rose to 50.07 lakh in 2017 and further to 50.74 lakh in 2018.[1] Latest released report showed that crime increased by 1.6% in 2019, i.e., it reached to 51.56 lakh.[2] Report further showed that crime against women increased by 7.3%, making it to a total of 4,05,861 cases. Although rape cases which went up from 32,500 in 2017 to 33,356 in 2018, i.e., 91 rapes per day, lower down to 87 rapes per day in 2019. The average is still a matter of concern. The group that has been suffering the most is "Women". One in three women gets affected by such gender-based violence at least once in their lifetime. As per Indian census 2001-2011, women population comprises of approx. 48.04% of India's total population.[3] Such disturbing trends in crime shows that about half of the population is facing such gender-based crimes. Women, despite their age, religion or caste, social well-being, economic stature, education, are being victimized of sexual violence and offenses like rape, harassment at workplace, domestic violence, kidnapping, suicide, dowry-related deaths. These crimes are somehow hindering their empowerment and progress, which directly affects the societal and economic development. World Bank report of 2018 claimed that loss in productivity due to violence against women, costs an estimate of 3.7% of country's GDP.[4] Thus, addressing this issue is of utmost importance.

With crime becoming more sophisticated and organized, certain approaches were adopted to regulate it. Several studies were carried out based on data analysis such as survey, quantitative analysis, self-report and structured interview to get some insights into the nature of victimization. Appreciable review was presented by Ellsworth in 2019,[5] comprising of 33 such studies, where he discussed the effect of street crime victimization on homeless adults.

Another effective approach to study the dynamics of crime is by formulating a mathematical model. There can be a case where carrying out actual experiment to know the behavior of real-world system can be expensive, impractical or impossible. For example, to study the dynamics of crime against women, it is impractical to do experiments on women. In that case, analysing the model provides insight on the parameters involved and allows making predictions about systems behavior. Such qualities of mathematical models make them more appropriate than carrying out actual experiment. Several mathematical models have been introduced in the past to study the crime dynamics, using different approaches like economical approach, epidemic approach, predator-prey modeling approach and spatio-temporal approach. Some of those models are discussed in the following section.

METHODS

Economical Approach

The economical approach is adopted in mathematical modeling for observing, understanding and making predictions about economic issues and problems.

In 1996, US Presidents Commission on Law Enforcement and Administration of Justice presented a report on crime in America to address the causes of crime and gave recommendations to prevent the same.[6] They created a task force on Science and Technology whose aim was to introduce simulation modeling to demonstrate how system models can be used to evaluate cost associated to crime and analyze the effects on crime rates. In 1968, Becker,[7] initiated an economic approach in crime modeling to measure the social loss caused by offenses and to further calculate the resources and punishments expenditure that minimize this loss. He developed a model considering the behavioral relations behind the costs of crime (involving white-collar crimes), by placing these relations into five different categories. In order to minimize social loss, he discussed the optimal conditions for decision variables p and f, representing probability of conviction and punishments respectively. The effects on the optimal values of p and f due to changes in the basic behavioral relations like the damage, cost, and supply-of-offenses functions were also analyzed. Further, several interesting implications of optimality conditions were illustrated via examples and at last, the optimal control of crime was determined through statistical and economic analysis.

To get a more realistic and dynamical approach towards optimal control of crime, Zhao *et al.,* have

proposed a model consisting of ordinary differential equations, assuming constant population.[8] The population was divided into five sub-classes, namely the non-poor class, the poor class, the criminal class, the jailed class and the recovered class. Through stability analysis, they studied the dynamics of poverty and crime which further helped in identifying the cost-effective strategies to control such crime. Since people immigrate for employment and other purposes, their assumption of constant population was not always the case in real world scenario.

Further, in 1998, Chiu has proposed an economic model relating burglary and crime distribution.[9] Assuming fixed incarnation time for burglars, and the fact that burglars choose their target houses by looking at the quality of house, he investigated the relation between level of crime and distribution of income. Through a result introduced by authors, the model analyzed that more regressive tax results in more crime and rich neighborhoods may result in lower crime rate. Using Lorenz and 'relative differential' comparisons of income distributions, he showed burglary crime level may be increased by increase in income inequality. Later, after analysing that houses close to burgled house have high probability of crime than those which are far away, Curtis *et al.,* have formulated a model relating crime and security.[10] They analyzed the role of technology on crime statistics by showing how crime probability decreases upon imposing security. Their model was based on an assumption of constant population, which is again not always the case in real world scenario. To fill this gap, in 2013 Shukla *et al.,* have introduced a model considering dynamic population, i.e. both immigration and emigration of susceptible and criminals were taken into account.[11] To study the impact of technology in combating crime, they considered five interacting variables, — susceptible population density, criminal density, removed criminal's density, crime burden density (proportional to criminal density) and the technology level used to control crime. They used stability theory to analyze inverse relation between crime and level of technology. The model also concluded that crime burden may reduce preventing illegal immigration.

Another type of crime that exists is financial crime, a property crime where an entity illicitly transforms the property's ownership for their own use and benefit. Financial crimes involve fraud, money laundering, identity theft, bribery, forgery. Some causes of financial crime involve greed, unemployment, unsatisfactory salary, laziness, quest to get rich. Importance of such crimes should not be minimized as they can lead to violent crimes and terrorism. Keeping this in mind, Akhani *et al.,* have introduced a mathematical model to analyze the dynamics of financial crime under some optimal control strategies and preventive measure.[12] The model was examined through stability analysis and reproduction number. Suitable Lyapunov functional was constructed to obtain crime free equilibrium and unique crime-present equilibrium, which were globally asymptotically stable for reproduction number less than unity and greater than unity respectively. Several parameters were tested against sensitivity analysis to determine their importance in combating financial crime. Further, the significance of implementing time dependent optimal control measures including public enlightenment campaign was discussed. They also implemented cost effective analysis which showed that the optimal use of public enlightenment campaign is the most effective and least costly strategy in combating financial crime.

Through the study of these models, it was observed that economical approach helped in measuring the social loss caused by several crimes and identifying some strategies, which help in combating crime and hence minimising the loss. The strategies included imposing security, preventing illegal immigration, use of public enlightenment campaign etc.

Epidemic Approach

Here crime is considered to be a social epidemic process. The approach is similar to the one used in mathematical biology to describe the spread or containment of epidemic in a population.

Such methodological approach was adopted by Campbell *et al.*[13] after the evidences provided by Glaeser *et al.*[14] in 1996 on the importance of social interaction. They considered it an important feature in their model i.e. susceptible are likely to commit crime upon coming in contact with criminals. They split the population into three

groups, namely non susceptible, susceptibles and active criminals and studied the effect of social interaction between the agents. Moreover, they assumed that any agent can be influenced by social interaction, not just a proportion of population, as assumed by Glaeser *et al.* Using same approach Gonzlez-Parra *et al.,* have presented a mathematical model by taking into account behavioral changes of individuals.[15] The total population was divided into six categories, namely susceptible, free criminals, convicted criminals, criminals arrested in jail, judges and police officers. They analyzed the system of six nonlinear ordinary differential equations by examining the obtained equilibria. The method of next generation matrix was used to compute threshold parameter for the extinction of criminality, which helped in understanding best policies for controlling crime. Furthermore, sensitivity analysis investigated some important parameters related to transition of susceptible, police officers and judges to criminals, lower criminality. Lastly through elasticity analysis, where the impact of the parameters on criminality was compared, it was observed that honest judges might reduce criminality in the society.

There can be a case where youth having high social and financial needs gets involved in illegal drug trafficking groups, controlled by adults. The social disease caused by this criminal interaction among youth and adult will spread like an epidemic in highly dense and metropolitan areas. To study the above-mentioned scenario, in 2019, Rivera-Castro *et al.,* have presented a model based on SIR disease dynamics by dividing the heterogeneous population into two groups namely youth and adult. Interaction between the group was governed by three mixing patterns which were proportionate, preferred and like with like. Basic reproduction number was derived to analyze the model and stability analysis was carried out for homogeneous mixing. Further, sensitivity analysis showed some insights about various parameters involved. The model concluded that involvement of kids in drug trafficking can be decreased if they do not interact with adult gangs, provided they don't have any previous gang involvement. Also, the best ways to reduce spread of gangs as suggested by like with like mixing is prevention and indulging in activities like arts, music, recreation, sports etc.

A corruption control model, having similar behavior as epidemiological model, was proposed by Athithan *et al.*[17] They divided the population into three classes, namely corruption susceptible, corruption infected and recovered from corruption class. The model was analyzed through stability analysis and threshold quantity R_0. Stability analysis showed the presence of two non-negative equilibria, where the first one was corruption free equilibrium and other was endemic equilibrium, which indicates presence of corruption. It was observed that corruption free equilibrium always exists and is stable for $R_0<1$and unstable otherwise. Whereas, endemic equilibrium exists only when $R_0>1$. To analyze the findings, numerical simulations were performed. Furthermore, they showed that including optimal control parameter such as self-cure rate in the model could minimize the corruption and the cost on reducing the corruption. It was also observed that providing psychological pressure through media/advertisements increased the self-cure rate and hence reduced the corruption in the society.

Mohammad and Roslan have provided a dynamical 2-Dimensional model to analyze the spread of crime system.[18] They referred the model introduced by McMillon et *al.,* in 2014,[19] with basic components being criminally active, not active and imprisoned population. The main objective of the paper was to analyze the model via stability analysis, bifurcation and reproduction number. Numerical simulations were carried out, as a result of which two equilibria were found, namely crime free equilibrium and crime equilibrium. Eigen values were calculated using Jacobian Matrix, whose sign represent whether the equilibrium point is stable, unstable or saddle. The results showed that crime free equilibrium was asymptotically stable while the crime equilibrium was unstable. Further, bifurcation analysis revealed that increasing a contagion parameter regarding criminal behavior in the model increase the number of criminally active and incarcerated ones. The model also showed that increase in the above-mentioned parameter also increases $R_0>1$ and hence increases crime rate.

In 2019, Ugwuishiwu *et al.,* have presented a deterministic model to analyze the dynamics of interaction between crime, criminality and

victimization in the population.[20] Two forms of rehabilitation having substantial effect on crime when implemented effectively were considered in the model, namely, a reformation program for criminal individuals and psychotherapy for victims. The proposed compartmental model was studied through stability analysis and two equilibria were obtained, namely crime free and persistent crime equilibrium. Their stability was computed with the help of reproduction number, a factor responsible for persistence of criminality or victimization. The model analyzed that crime free equilibrium was globally-asymptotically stable when $R_0<1$, whereas unique endemic equilibrium was locally-asymptotically stable for $R_0>1$. Also, forward bifurcation was discussed using Centre Manifold theory. At last, the consequences of both effective and ineffective implementation on the rehabilitation forms were discussed. It was observed that non-criminalisation of victims is less effective in containing crime than the prevention of repeat victimization. A positive impact was observed from the removal of criminals either through quitting or death.

Summarizing the above models, it can be observed that the epidemic approach provided a better understanding on the influence of social interaction on spread of crime. The models also discussed several parameters to lower criminality, say honest judges, indulging susceptible in other activities like music and sports, reducing corruption by increasing self-cure rate, implementing rehabilitation forms, reducing criminal etc.

Predator-Prey Modeling Approach

Another approach to study the dynamics of crime is through Predator-Prey Modeling. Such approach helps in understanding the interaction of populations under natural environment.

Nuno *et al.,* consider such modeling[21] to discuss the interaction between three classes. Namely, owner, criminal and security guard. They considered owners to be the prey, criminals to be the predators of owners, and the security guards were predators of both owners and criminals. In this triangle model, all classes were competing for same resources. Later in 2016, Sooknanan *et al.,* then presented a modified predator-prey model in combination with an infectious disease (eco-epidemiological model), to examine the relationship between police (predator) and gang members (prey).[22] The model dealt with the corruption of police officers caused by gang members and other corrupted police officers. Further, the model analyzed various crime fighting strategies and policy changes. By assuming constant number of police officers, several behaviors of the model were interpreted through stability analysis. Response of gang members to various crime fighting strategies and policy changes were discussed. At last, they identified tripping points, which could lead to disappearance of such gangs and corrupted police officers from the population.

Motivated by both predator-prey and epidemic approach, Abbas *et al*., have introduced a model to examine the interaction between criminal and non-criminal population.[23] They changed the growth of non-criminal population from exponential to logistic. The logistic model showed that density of criminal population vanishes as carrying capacity goes below its threshold value (R_0). Whereas, it increases slowly when carrying capacity crosses its threshold value. In terms of financial conditions, this meant that rate of increase of crime can be controlled by improving the living standards. Logistic model was further modified to law enforcement model, which revealed that rate of crime decreases as enforcement parameter increases. Moreover, once the enforcement parameter reaches its threshold value, the population becomes crime free. Occurrence of saddle node bifurcations in law enforcement model was also discussed. However, their model did not incorporate the fact that under proper counseling criminals might recover and leave crime temporarily or permanently.

In 2019, Srivastav *et al.,* have provided an extended model by introducing a new class namely, recovered class.[24] They assumed that criminals might move to recovered class upon proper law enforcement and proper counseling. Also, they assumed that the negligible effects of counseling and unpredictable nature of human behavior could lead to a flow from recovered class to criminal and non-criminal class. The non-linear mathematical model was analyzed by computing basic reproduction number, to determine the persistence of crime. The existence and stability of different equilibria were discussed in detail. Further, the

authors discussed that improper law and orders can delay in flow of criminal to recovered class. Thus, causing delay in catching criminals. The fact was incorporated in the model to provide extended delay differential equation model. Stability analysis on the delay model revealed that large delays could cause periodic oscillations and hence further make it challenging to control the spread of crime. Lastly, the deterministic model was converted to stochastic model and their results were compared using numerical simulation, observing a lower level of criminal population in stochastic than in deterministic model.

The models based on predator-prey approach divide the population into two parts- predator and prey and study the relationship between them. Further, along with epidemic approach, the models showed that criminals could be recovered upon proper law enforcement and counseling.

Spatio-Temporal Approach

There is a line of research which talks about spatial and temporal dynamics of crime. In the last two decades, many researchers have focused on the spatio-temporal modeling of crime dynamics.[25-31] Empirical observation showed that crime is not uniformly distributed within time and space. Some areas are considerably safe whereas some may experience dense clusters of crime. Also, repeat victimization was observed over short intervals. Such spatial temporal clusters having high intensity crime are known as hotspots. Identification of such hotspots enables to analyze the conditions which leads to occurrence of crime. Several theories have been provided to understand the emergence of these hotspots, connection of their size and features with behavior of victims, offenders, law enforcements agents and local geography. It was observed that structure of urban environment involving features like population density, traffic volume, vacant property impacted crime patterns.[32-37]

Under certain conditions, crime hotspots may emerge, dissipate or diffuse. To examine the essential dynamics of these hotspots, in 2008, Short *et al.*, have introduced a quantitative 2-D model by focusing on simplest urban crime, i.e., residential burglary.[25] They started with a discrete system and derived a continuum model. Numerical simulations revealed that the two models were in good approximation under large criminal population. Lastly, stability analysis was done to determine the parameter values which lead to formation of stable hotspots. Further, in 2010, Short *et al.,* have included repeat victimization in their work and presented a model to study the repeat and near repeat burglary effects.[26] They concluded that such repeat and non-repeat victimization might lead to formation of crime hotspots and therefore might be analyzed for designing crime prevention strategies.

As a result of crime hotspot policing, it was observed that such hotspots may suppress, dissipate or displace. To study this scenario, in 2009, Short *et al.,* have presented reaction diffusion model.[27] Such models help in investigating the formation of crime patterns and the impact of alternative policing strategies on crime hotspot stability. They classified hotspots as supercritical and subcritical, where the former is a result of small spikes in crime which creates a local crime wave and latter arises due to large spike in crime. Their method showed that subcritical hotspots might be permanently destroyed whereas supercritical hotspots could only be displayed following a characteristic spatial pattern.

Thus, it can be concluded that spatio-temporal approach helps in identifying crime hotspots and further analysing such hotspots aids in designing crime prevention strategies.

DISCUSSION & CONCLUSION

Mathematical models can be used to guide decision-making, develop policies or to evaluate specific strategies aimed at reducing crime. In this review paper, we have discussed several approaches that can be adopted for formulation of crime models. One such approach was Economical approach, where the mathematical models were formulated to measure the social loss caused from offenses and to further identify the cost-effective strategies for the control of crime. As crime is considered a social epidemic, Epidemic approach was used, where crime was considered to spread in the population like an infectious disease. The authors assumed that susceptible were likely to commit crime upon coming in contact with criminals. Through such models, the impact of behavioral changes and policy implementations was discussed. Also, a deterministic model was

analyzed to identify the consequences of criminals and criminality on victimization. Further, we discussed about the models showing interactions among classes using predator-prey modeling approach, where one class was considered as prey and other as predator. This approach was combined with epidemic approach to formulate a model examining the relation between police and gang members. Such model helped in identifying the tripping points, which could lead to disappearance of gangs and corrupted police officers from the population. Interaction between criminal and non-criminal population was also studied through the combination of these two approaches. It was observed that criminals might move to recovered class upon proper law enforcement and proper counseling. Furthermore, empirical observations indicated that crime was not uniformly distributed but used to vary with time and location. Such spatio-temporal approach was adopted by several authors to identify the clusters having high intensity crime (known as *hotspots*).

In all the above reviewed models, we noted that mathematical models describing the dynamics of crime against women are given least importance. Several methods and approaches have been studied in this review, which are expected to give hints regarding the formation of models that would be helpful in controlling the spread of crime against women. This will open a window for counselors, bureau of police research and development and other law enforcement agencies to predict the trends in the crime against women and prepare the optimal control and prevention policies. IJFMP

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